# Long-range magnetic ordering of $S=1 / 2$ linear trimers in $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}, \mathrm{Sr}$, and Pb$)$ 

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#### Abstract

Magnetic properties of $S=1 / 2$ linear trimer cluster compounds $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}, \mathrm{Sr}$, and Pb$)$ were investigated. Magnetic susceptibility data for the three compounds showed that paramagnetic copper spins form trimers with the total spin of $1 / 2$ below about 45 K . Specific heat and magnetization measurements indicated that the trimer clusters undergo ferromagnetic long-range ordering at $T_{\mathrm{C}}=0.91 \mathrm{~K}$ for $A=\mathrm{Ca}$ and antiferromagnetic long-range ordering at $T_{\mathrm{N}}=0.91 \mathrm{~K}$ for $A=\mathrm{Sr}$ and $T_{\mathrm{N}}=1.26 \mathrm{~K}$ for $A=\mathrm{Pb} . A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ exhibited $1 / 3$-magnetization plateau at least up to magnetic field of 55 T at 1.3 and $4.2 \mathrm{~K} . A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ with $A=\mathrm{Sr}$ and Pb showed a spin-flop transition near 0.03 T in the antiferromagnetic state at 0.08 K . Specific heat data at magnetic fields clearly showed broad maxima at low temperatures due to the finite intra-chain interaction in one-dimensional arrays of the trimers. (C) 2005 Elsevier Inc. All rights reserved.


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## 1. Introduction

Low-dimensional quantum magnets have been a subject of intense studies in recent years. In particular, zero-dimensional (cluster) systems with spin $(S)$ of $1 / 2$ is considered to be interesting because the quantum effects are much pronounced.

Magnetic interactions ( $J_{1}$ and $J_{2}$ ) between copper atoms in two isostructural compounds $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ ( $A=\mathrm{Ca}$ [1] and Sr [2]), belonging to space group $P 2_{1} / c$, are shown on Fig. 1a. They can be presented by the onedimensional (1D) array of trimer clusters, where the trimers possessing the total spin of $1 / 2$ are connected on one direction. In other words, it is a kind of 1 D ferrimagnet. There are two copper sites. The Cu 1 site

[^0]has a square planar coordination and the Cu 2 site has a distorted square pyramidal coordination. Cu 1 and Cu 2 are connected with each other through a common oxygen atom. By analyzing the magnetic susceptibility and specific heat ( $T=1.8-15 \mathrm{~K}$ ) data [3-5], the intratrimer exchange constant $\left(J_{1}\right)$ was estimated to be about 100 K , while the inter-trimer exchange constant $\left(J_{2}\right)$ about 3 K . Thus, $J_{1}$ is much larger than $J_{2}$ and $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ is a model system of the $S=1 / 2$ linear Heisenberg antiferromagnetic trimer [6]. $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ was reported to order ferromagnetically at 0.8 K , while $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ exhibits an antiferromagnetic ordering at 0.9 K [4]. Thus, the inter-chain (Fig. 1b) interactions are also expected to be very small. Magnetization measurements up to 40 T showed the existence of $1 / 3$-magnetization plateau [7]. In addition, magnetic susceptibility curves of $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}$ and Sr$)$ exhibit a clear signature of 1 D ferrimagnets, i.e., a rounded minimum of $\chi T$ near 25 K and strong increase of $\chi T$ at the lower temperatures $[3,4]$.
(a)



Fig. 1. (a) Projection of the 1 D copper trimeric chain on the $b c$ plane in $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ and $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$. The intra-trimer ( $J_{1}$ : black stick) and intra-chain ( $J_{2}$ : white stick) interactions are shown. (b) Projection of the structures of $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ and $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ along the $b$ axes: arrangement of the 1D copper chains. Unit cells for $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ and $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ are shown by solid lines and lattice parameters $(a, b$, and $c$ in $\AA$ and $\beta$ in deg) are given. For $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$, the monoclinic cell ( $a^{\prime}, c^{\prime}$, and $\beta^{\prime}$ ) in another setting $(I 2 / a)$ is shown by the broken line.

Recently, a new compound $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$, crystallizing in space group $C 2 / c$, was reported [8]. The lattice parameters of $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}\left(a_{\mathrm{Sr}}, b_{\mathrm{Sr}}, c_{\mathrm{Sr}}\right.$, and $\left.\beta_{\mathrm{Sr}}\right)$ have the following relation with those of $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}\left(a_{\mathrm{Pb}}^{\prime}\right.$, $b^{\prime}{ }_{\mathrm{Pb}}, c^{\prime}{ }_{\mathrm{Pb}}$, and $\left.\beta^{\prime}{ }_{\mathrm{Pb}}\right): a_{\mathrm{Sr}} \approx 0.5 c^{\prime}{ }_{\mathrm{Pb}}, b_{\mathrm{Sr}} \approx b_{\mathrm{Pb}}^{\prime}, c_{\mathrm{Sr}} \approx a^{\prime}{ }_{\mathrm{Pb}}$, and $\beta_{\mathrm{Sr}} \approx \beta^{\prime}{ }_{\mathrm{Pb}}$ (Fig. 1b). The coordination polyhedra of Cu and $\mathrm{Sr}(\mathrm{Pb})$ atoms are the same in $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ and $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$, that is, $\mathrm{Cu}^{2} \mathrm{O}_{4}, \mathrm{Cu} 2 \mathrm{O}_{5}, \mathrm{Sr}(\mathrm{Pb}) \mathrm{O}_{6}$, and $\mathrm{Sr} 2(\mathrm{~Pb} 2) \mathrm{O}_{9} . \mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ has the similar 1D chains of the copper trimers as shown on Fig. 1a. However, the arrangement of the 1 D chains is different in $\mathrm{Sr}_{3} \mathrm{Cu}_{3}$ $\left(\mathrm{PO}_{4}\right)_{4}$ and $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$. While the projection of the 1 D chains along the $b$ axes is the same in $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ and $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ (Fig. 1b), the adjacent 1D chains have different orientations as can be seen on the projection along the $\boldsymbol{a}_{\mathrm{Sr}}$ axis in $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ and along the $\left(\boldsymbol{a}_{\mathrm{Pb}}+\boldsymbol{c}_{\mathrm{Pb}}\right)$ vector in $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ (Fig. 2). Note that $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ and $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ prepared by the solid state method are isotypic with each other [2]. $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ prepared by hydrothermal methods is topologically similar but not isotypic with $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ [8]. Magnetic properties of $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ have not been investigated yet.

In this work, we report on the properties of $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ studied by magnetic susceptibility, spe-


Fig. 2. Projection of the 1D copper trimeric chains in (a) $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ with the $\boldsymbol{a}_{\mathrm{Sr}}$ axis perpendicular to this figure and (b) $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ with the $\left(\boldsymbol{a}_{\mathrm{Pb}}+\boldsymbol{c}_{\mathrm{Pb}}\right)$ vector perpendicular to this figure. Sr and Pb atoms are also shown.
cific heat, and magnetization measurements. We also measured magnetization curves up to 55 T and specific heat data in a wider temperature range and in magnetic fields for the related compounds $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}$ and Sr ).

## 2. Experiment section

The samples $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}, \mathrm{Sr}$, and Pb$)$ were prepared from stoichiometric mixtures of CuO , $\mathrm{NH}_{4} \mathrm{H}_{2} \mathrm{PO}_{4}, \mathrm{CaCO}_{3}, \mathrm{SrCO}_{3}$, and PbO by the solid state method. The corresponding mixtures were heated very slowly in air from room temperature to 770 K (kept for 20 h ), reground, pressed into pellets at $200 \mathrm{kgf} / \mathrm{cm}^{2}$, and allowed to react at 1183 K for $A=\mathrm{Ca}$ and Sr and at 993 K for $A=\mathrm{Pb}$ on Pt plates for 100 h with four intermediate grindings. X-ray powder diffraction (XRD) data collected with a RIGAKU RINT 2500 diffractometer ( $2 \theta$ range of $8-60^{\circ}$, a step width of $0.02^{\circ}$, and a counting time of $1 \mathrm{~s} /$ step) showed that the samples were monophasic. Note that the differential thermal analysis (DTA) data showed that $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}$, Sr , and Pb ) melts incongruently at 1333,1281 , and 1036 K , respectively. The DTA curves also indicated the existence of a reversible phase transition in $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ ( $A=\mathrm{Ca}$ and Sr ) just below (about 10 K ) the decomposition temperatures, i.e., at 1324 and 1273 K for $A=\mathrm{Ca}$ and Sr , respectively.

Magnetic susceptibility $(\chi=M / H)$ measurements were performed on a Quantum Design SQUID magnetometer (MPMS XL) between 2 and 400 K in applied fields of 1000 Oe for $A=\mathrm{Ca}$ and Sr and 100 Oe for $A=\mathrm{Pb}$ under both zero-field-cooled (ZFC) and fieldcooled (FC: measured in a magnetic field from 400 to $2 \mathrm{~K})$ conditions. High-field magnetization data were taken at 1.3 and 4.2 K in a pulsed magnetic field up to 55 T and at 0.08 K up to 25 T by an induction method using a multilayer pulse magnet at KYOKUGEN, Osaka University. Specific heat, $C_{\mathrm{p}}(T)$, of $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ was recorded between 0.45 and 300 K (on cooling) at zero magnetic field and between 1.8 and 100 K at magnetic fields of 1,3 , and 5 T by a pulse relaxation method using a commercial calorimeter (Quantum Design PPMS $)$. For $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}$ and Sr$)$, the $C_{\mathrm{p}}(T)$ data were taken between 0.45 and 170 K at zero magnetic field and between 1.8 and 50 K at magnetic fields of 1,3 , and 5 T .

## 3. Results and discussion

The temperature dependence of the inverse magnetic susceptibility, $\chi^{-1}$, and $\chi T$ for $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ is displayed in Fig. 3. No noticeable difference is found between the ZFC and FC curves. The $\chi^{-1}$ vs. $T$ curve exhibits a curvature near 45 K . Between 150 and 400 K , the $\chi^{-1}$ vs. $T$ curve is fitted by the modified Curie-Weiss equation
$\chi(T)=\chi_{0}+\frac{C}{T-\theta}$
with the temperature independent term, $\chi_{0}$, of $-2.53(3) \times 10^{-4} \mathrm{~cm}^{3} \mathrm{~K} / \mathrm{Cu} \mathrm{mol}$, the Curie constant, $C$, of $0.496(2) \mathrm{cm}^{3} \mathrm{~K} / \mathrm{Cu} \mathrm{mol}$, and the Weiss constant, $\theta$, of $-57.0(7) \mathrm{K}$. These values show that each Cu ion has spin- $1 / 2$ with $g$ value of 2.30 and these interact antiferromagnetically with each other. Between 7 and 30 K , the fitting to Eq. (1) gave $\chi_{0}=1.0(4) \times 10^{-4} \mathrm{~cm}^{3}$ $\mathrm{K} / \mathrm{Cumol}, \quad C=0.1585(12) \mathrm{cm}^{3} \mathrm{~K} / \mathrm{Cumol}$, and $\theta=$ $2.77(5) \mathrm{K}$. The ratio between the Curie constants at the


Fig. 3. Inverse magnetic susceptibilities, $\chi^{-1}$, and the product $\chi T$ against temperature, $T$, for $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$. The solid line for the $\chi^{-1}(T)$ curve is the fit to Eq. (2).
high and low temperature regions, $C_{\mathrm{HT}} / C_{\mathrm{LT}}$, is 3.13 . The small Curie constant below 45 K indicates that paramagnetic copper spins form trimer clusters because of relatively strong intra-trimer antiferromagnetic interactions and each trimer has a total spin $\left(S_{\text {total }}\right)$ of $1 / 2$. The $\chi T$ vs. $T$ curve of $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ is similar to those of $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}$ and Sr$)[3,4]$ and demonstrates the typical features of 1 D ferrimagnets, i.e., a rounded minimum near 28 K and then a strong increase at the lower temperatures (Fig. 3).

At the whole temperature range of $2-400 \mathrm{~K}$, the $\chi^{-1}$ vs. $T$ curve is fitted by the model of the isolated $S=1 / 2$ Heisenberg linear trimers [5]

$$
\begin{align*}
& \chi(T) \\
& =\chi_{0}+\frac{N g^{2} \mu_{\mathrm{B}}^{2}}{12 k_{\mathrm{B}} T} \frac{1+\exp \left(J_{1} / k_{\mathrm{B}} T\right)+10 \exp \left(-J_{1} / 2 k_{\mathrm{B}} T\right)}{1+\exp \left(J_{1} / k_{\mathrm{B}} T\right)+2 \exp \left(-J_{1} / 2 k_{\mathrm{B}} T\right)} \tag{2}
\end{align*}
$$

where $N$ is the Avogadro's number, $g$ the spectroscopic splitting factor ( $g$-factor), $\mu_{\mathrm{B}}$ the Bohr magneton, $k_{\mathrm{B}}$ the Boltzmann's constant, and $J_{1}$ the intra-trimer exchange constant (Fig. 1a) with the Hamiltonian formulated as $H=J_{1}\left(S_{3 i} S_{3 i+1}+S_{3 i} S_{3 i-1}\right)$. The fitted parameters are $\chi_{0}=-2.97(12) \times 10^{-4} \mathrm{~cm}^{3} \quad \mathrm{~K} / \mathrm{Cumol}, \quad g=2.291(13)$, and $J_{1} / k_{\mathrm{B}}=106(2) \mathrm{K}$ for $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$. The $J_{1} / k_{\mathrm{B}}$ value determined from the magnetic susceptibility data is close to that found from inelastic neutron scattering experiments ( $J_{1} / k_{\mathrm{B}}=106 \mathrm{~K}$ ) [6].

The fitting of our magnetic susceptibility data ( $\chi^{-1}$ vs. $T$ ) for $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}$ and Sr$)$ by Eq. (2) yields $g=2.32(2)$ and $J_{1} / k_{\mathrm{B}}=127(3) \mathrm{K}$ for $A=\mathrm{Ca}$ and $g=$ $2.27(2)$ and $J_{1} / k_{\mathrm{B}}=126(3) \mathrm{K}$ for $A=\mathrm{Sr}$. The $g$ values are in excellent agreement with those determined from the ESR measurements $(g=2.3)$ [7]. However, the $J_{1} / k_{\mathrm{B}}$ values are slightly larger than those obtained from inelastic neutron scattering experiments $(110 \mathrm{~K}$ for $A=\mathrm{Ca}$ and 117 K for $A=\mathrm{Sr}$ ) [6] .

Specific heat data between 0.45 and 15 K for $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ are given in Fig. 4. The $\lambda$-type peak is


Fig. 4. The $C_{\mathrm{p}}$ vs. $T$ curves for $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ at $0,1,3$, and 5 T . The parameters of the broad maximum $\left(T_{\max }(\mathrm{K})\right.$ and $\left.C_{\max }\left(\mathrm{J} \mathrm{K}^{-1} \mathrm{~mol}^{-1}\right)\right)$ at 1 T are given. Solid line is the estimated lattice contribution. Inset gives the $C_{\mathrm{p}}$ vs. $T$ curve between 0.45 and 300 K .


Fig. 5. The $C_{\text {p }}$ vs. $T$ curves for (a) $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ and (b) $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ at $0,1,3$, and 5 T . The parameters of the broad maximum $\left(T_{\max }\right.$ and $C_{\max }$ ) at 1 T are given. Solid line is the estimated lattice contribution.
seen at $1.26(3) \mathrm{K}$ indicating long-range magnetic ordering in $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$. In $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}$ and Sr$)$, the $\lambda$-type peak is observed at $0.91(3) \mathrm{K}$ for both compounds (Fig. 5). This temperature is close to the temperatures of long-range ordering reported in the literature [4]. However, the peaks are rather unusual. For example, the $C_{\mathrm{p}}$ values for $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ start to increase gradually at about 7 K with decreasing the temperature and then a sharp peak indicating the longrange ordering appears at 0.91 K . It seems that the peak has two components. It was predicted that the specific heat of the magnetic system given on Fig. 1a has intrinsic double-peak structure [3,9]. One is the Schott-ky-type peak corresponding to the intra-trimer interaction at about $0.4 J_{1}$. The temperature position and the value of the second anomaly depend on the ratio $J_{2} / J_{1}$. When $J_{2} / J_{1}$ is small enough, the clear second anomaly should be observed at low temperatures on magnetic specific heat, $C_{\mathrm{m}}$. In the present data for $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ ( $A=\mathrm{Ca}, \mathrm{Sr}$, and Pb ) taken in zero magnetic field, the second anomalies overlap with the $\lambda$-type peaks (Figs. 4 and 5). These anomalies are more pronounced in the data taken in the magnetic fields.
Unfortunately, the numerical calculations of $C_{\mathrm{m}}$ for the system given in Fig. 1a were done only for antiferromagnetic $J_{2}$, at zero magnetic field, and for $J_{2} /$ $J_{1} \geqslant 0.1[3,9]$. However, the intra-chain interaction in the 1 D array of the trimers ( $J_{2}$ ) may be ferromagnetic as indicated by the positive Weiss constant at the low temperature region. The numerical calculation are
desirable in magnetic fields and for $\left|J_{2}\right| / J_{1}<0.1$ because they will help to estimate $J_{2}$ from the specific heat data. Note that such field dependence of specific heat as shown in Figs. 4 and 5 is observed in 1D $S=1 / 2$ Heisenberg ferromagnets [10].

The lattice contribution to the specific heat in $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}, \mathrm{Sr}$, and Pb$)$ was estimated between 10 and 14 K using the expression $C_{1}=\beta_{1} T^{3}+$ $\beta_{2} T^{5}\left(\beta_{1}=1.05(3) \times 10^{-3} \mathrm{~J} \mathrm{~K}^{-4} \mathrm{~mol}^{-1}, \beta_{2}=5(2) \times 10^{-7}\right.$ $\mathrm{J} \mathrm{K}^{-6} \mathrm{~mol}^{-1}$ for $A=\mathrm{Ca}, \quad \beta_{1}=1.703(10) \times 10^{-3} \mathrm{~J} \mathrm{~K}^{-4}$ $\mathrm{mol}^{-1}, \beta_{2}=1.1(6) \times 10^{-7} \mathrm{~J} \mathrm{~K}^{-6} \mathrm{~mol}^{-1}$ for $A=\mathrm{Sr}$, and $\beta_{1}=4.15(7) \times 10^{-3} \mathrm{~J} \mathrm{~K}^{-4} \mathrm{~mol}^{-1}, \quad \beta_{2}=1.3(4) \times 10^{-6}$ $\mathrm{J} \mathrm{K}^{-6} \mathrm{~mol}^{-1}$ for $A=\mathrm{Pb}$ ) assuming magnetic contribution is negligible at these temperatures. The magnetic specific heat is not accurately zero, but it should be quite small because the influence from two Schottky-type anomalies is the smallest in this temperature range [3]. The $\beta_{1}$ values give Debye temperatures, $\Theta_{\mathrm{D}}=\left(234 N k_{\mathrm{B}} /\right.$ $\left.\beta_{1}\right)^{1 / 3}$, of 123,104 , and 78 K for $A=\mathrm{Ca}, \mathrm{Sr}$ and Pb , respectively. The magnetic entropy, $S_{\mathrm{m}}$, associated with the long-range ordering is shown in Fig. 6. $S_{\mathrm{m}}$ saturates at about $5.11,5.24$, and $4.96 \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}$ for $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ with $A=\mathrm{Ca}, \mathrm{Sr}$, and Pb , respectively. The entropy gained below the long-range ordering temperatures ( $T_{\mathrm{N}}$ or $T_{\mathrm{C}}$ ) is $31 \%, 35 \%$, and $42 \%$ of the saturated values for $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}, \mathrm{Sr}$, and Pb$)$, respectively. The entropy gained below $T_{\mathrm{N}}\left(T_{\mathrm{C}}\right)$ gives a barometer for the inter-chain interaction. For a good 1D system, less than about $10 \%$ of the entropy is gained below $T_{\mathrm{N}}\left(T_{\mathrm{C}}\right)$ and the remaining is gained slowly up to the temperature corresponding to $J$. Judging from above values, the inter-chain interactions are not negligible for the present systems compared with the intra-chain interaction $\left(J_{2}\right)$. This argument is consistent with the relatively large $T_{\mathrm{N}} / J_{2}$ values. The ground state of the linear trimer is a doublet with $E(0)=-J_{1}$ and $S_{\text {total }}=1 / 2[11]$, that is, there is one Kramers doublet. The entropy change owing to the long-range ordering of the trimers is therefore expected to be $R \ln 2 \approx$ $5.76 \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}$. The experimental $S_{\mathrm{m}}$ is smaller than the expected value probably due to the overestimation of the lattice contribution. Note that the reduced value


Fig. 6. The $C_{\mathrm{p}} / T$ vs. $T$ and $S_{\mathrm{m}}$ vs. $T$ curves at zero magnetic field for $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}, \mathrm{Sr}$, and Pb$)$.


Fig. 7. The $M$ vs. $H$ curves at $0.08,1.3$, and 4.2 K for $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ between 0 and 15 T . Inset gives the $M$ vs. $H$ curve at 4.2 K up to 55 T .
of $S_{\mathrm{m}}$ was also reported for the antiferromagnetic ordering of $S=1 / 2$ triangles in $\mathrm{La}_{4} \mathrm{Cu}_{3} \mathrm{MoO}_{12}$ [12].

The magnetization curves for $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}$, Sr , and Pb ) saturate at about $1.15-1.20 \mu_{\mathrm{B}} / \mathrm{mol}$ (Fig. 7). This value is one-third of the expected saturation value, $M_{\mathrm{s}}$, of $3.45 \mu_{\mathrm{B}} / \mathrm{mol}$ for $S=1 / 2$ and $g=2.3$ [7]. The observed intermediate quantum state corresponds to $S=1 / 2$ per trimer. The $1 / 3$ magnetization plateau persists up to 55 T at 1.3 and 4.2 K . The excitation energy from $S_{\text {total }}=1 / 2$ to $S_{\text {total }}=3 / 2$ is $1.5 J_{1}$ for the linear trimer model [11]. Applying $J_{1} / k_{\mathrm{B}}=106 \mathrm{~K}$, estimated from the susceptibility data for $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$, it is expected that the spin flip will be observed at $H_{\mathrm{sf}} \approx 103 \mathrm{~T}$.

No hysteresis was found on the $M$ vs. $H$ curves at 0.08 K for $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Sr}$ and Pb$)$, i.e., below the temperatures of long range ordering (Fig. 8). The $M$ vs. $H$ curves also pass through the origin. These facts give us evidence to believe that there is no net magnetic moment in the ground state of $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Sr}$ and Pb ), that is, they order antiferromagnetically. At about 0.03 T , the slope change of the $M$ vs. $H$ curves was detected in $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Sr}$ and Pb$)$ due to a spin flop transition in the antiferromagnetic state. The small value of the magnetic field for the spin flop transition reflects the small value of the inter-chain interaction.

The $M$ vs. $H$ curve of $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ was different from those of $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Sr}$ and Pb$)$ and no spin-flop transition was observed in $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$. Our data confirmed the different behavior of $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ at very low temperatures in comparison with $\mathrm{Sr}_{3} \mathrm{Cu}_{3}$ $\left(\mathrm{PO}_{4}\right)_{4}$ reported in the literature [4]. Taking into account the results of $a c$ susceptibility measurements [4] and our $M$ vs. $H$ curves, $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ orders ferromagnetically at $T_{\mathrm{C}}=0.91 \mathrm{~K}$. The $M$ vs. $H$ curves indicates that $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ behaves as a very soft ferromagnet because the $M$ vs. $H$ curves passed through the origin within the system resolution. The different behavior of $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ and $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ is interesting because they are isotypic with each other and have the same arrangement of the 1D trimeric chains.


Fig. 8. The $M$ vs. $H$ (black diamonds) and $\mathrm{dM} / \mathrm{dH}$ vs. $H$ (circles) curves at 0.08 K for (a) $\mathrm{Ca}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$, (b) $\mathrm{Sr}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$, and (c) $\mathrm{Pb}_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}$ between -0.5 and 0.5 T . These $M$ vs. $H$ curves were measured from 25 to about -3 T .

In conclusion, we investigated magnetic properties of $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Ca}, \mathrm{Sr}$, and Pb$)$. The paramagnetic moment decreases to $S=1 / 2$ per trimer below 45 K due to the antiferromagnetic intra-trimer interaction. The trimers exhibit ferromagnetic long-range ordering at $T_{\mathrm{C}}=0.91 \mathrm{~K}$ for $A=\mathrm{Ca}$ and antiferromagnetic longrange ordering at $T_{\mathrm{N}}=0.91 \mathrm{~K}$ for $A=\mathrm{Sr}$ and $T_{\mathrm{N}}=1.26 \mathrm{~K}$ for $A=\mathrm{Pb}$. The intermediate $1 / 3$-magnetization plateau at 1.3 and 4.2 K persists up to 55 T . Below $T_{\mathrm{N}}$, the clear spin flop transition is detected at low magnetic field in $A_{3} \mathrm{Cu}_{3}\left(\mathrm{PO}_{4}\right)_{4}(A=\mathrm{Sr}$ and Pb$)$. We observed strong field dependence of specific heat due to the finite intra-chain interaction $\left(J_{2}\right)$ in the 1 D array of the trimers.

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